P.G. sem-2nd paper-IV, unit-IV Complex integration Dr. S.D. Singh

④ If $0 < |z-1| < 2$ then express $f(z) = \frac{z}{(z-1)(z-3)}$ in a series of positive and negative powers of $(z-1)$.

रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

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3 गुरु Let $u = z-1$ so $0 < |u| < 2$

or $\frac{|u|}{2} < 1$

$\therefore f(z) = \frac{z}{(z-1)(z-3)}$

$= \frac{A}{z-1} + \frac{B}{z-3} = \frac{A(z-3)+B(z-1)}{(z-1)(z-3)}$

$\therefore z = A(z-3) + B(z-1)$

Let $z=3$,

$3 = A(0) + B(2) \quad \therefore B = \frac{3}{2}$

4 शुक्र $z=1, \quad 1 = A(-2)+B(0) \therefore A = -1/2$

$\therefore f(z) = -\frac{1}{2(z-1)} + \frac{3}{2(z-3)}$

$= -\frac{1}{2}\cdot\frac{1}{u} + \frac{3}{2} \times \frac{1}{z-1-2}$

$= -\frac{1}{2u} + \frac{3}{2(u-2)} = -\frac{1}{2u} + \frac{3}{-2\times 2(1-\frac{u}{2})}$

$= -\frac{1}{2u} - \frac{3}{4}\left(1-\frac{u}{2}\right)^{-1}$

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$$= \frac{-1}{2u} - \frac{3}{4}\left\{1 + \frac{u}{2} + \frac{u^2}{4} + \frac{u^3}{2^3} + \cdots + \left(\frac{u}{2}\right)^n + \right.$$ 5 शनि

$$= -\frac{3}{4}\sum_{n=0}^{\infty}\left(\frac{u}{2}\right)^n - \frac{1}{2u}$$

$$= -\frac{3}{4}\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^n} - \frac{1}{2(z-1)}$$ Ans

(5.) Prove that

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \cdots$$

$$= |z-1| < 1$$

Let $f(z) = \log z$ 6 रवि

By Taylor's theorem,

$$f(z) = \sum_{n=0}^{\infty}\frac{(z-a)^n}{\lfloor n} f^n(a)$$

$$= f(a) + \frac{(z-a)}{\lfloor 1} f'(a) + \frac{(z-a)^2}{\lfloor 2} f''(a) + \cdots$$

Taking $a = 1$

$$f(z) = f(1) + \frac{z-1}{\lfloor 1} f'(1) + \frac{(z-1)^2}{\lfloor 2} f''(1) + \cdots$$ (1)

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7 सोम

Now $f(1) = \log(1) = 0$

$\because f(z) = \log z$

$\therefore f'(z) = \frac{1}{z} = z^{-1}$

$f''(z) = -z^{-2}$

$f'''(z) = 2z^{-3}$

$\therefore f(1) = \log 1 = 0, \quad f'(1) = \frac{1}{1} = 1$

$f''(1) = -1, \quad f'''(1) = 2$

From (1)

$$f(z) = f(1) + (z-1)f'(1) + \frac{(z-1)^2}{\lfloor 2} f''(1) + \cdots$$

8 मंगल

$$\log z = 0 + (z-1)(1) - \frac{(z-1)^2}{\lfloor 3} + \cdots$$

Proved